Pivot-Wheel Drive – *Crab with a Twist!*

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4-Wheel Independent Pivot-Wheel Drive describes a 4wd drive-train in which each of the (4) wheels are independently driven and may be independently pivoted for steering purposes. The design offers the potential for excellent drive-train performance and a solution to conventional (tank) drive-train design constraints. The design also brings some clear design and control challenges.

This arrangement provides the possibility to operate in several different modes. One of these modes is Crab mode. In Crab Mode, all wheels steer together and drive at a common speed thereby steering the robot in any direction on the 2-d playing surface (true 2-d drive). As described above, this mode does not allow overt control of chassis orientation.

Gary Deaver, a Team 1640 Mentor, made the case that a Pivot Drive robot with 4 independently driven and steered wheels was not intrinsically constrained to typical Crab behavior and that it should be feasible to overtly control chassis orientation. Furthermore, from a human interface standpoint, a 3-axis joystick which includes a twist axis as the 3rd control axis would provide an intuitive means of human control over a Crab twist function.

The ability to control chassis orientation in Crab Mode provides an obvious operating advantage.

This paper addresses how to accomplish such "twist" behavior from a Pivot and Drive standpoint.

The Basic Crab Chassis

The Basic Crab Chassis is shown in Figure 1. There are 4 Pivot Wheels set on an *l x w* wheelbase. Wheel numbering here is consistent with the earlier *"Programming a Pivot-Wheel Drive"* paper of 9-August-2009. A chassis Center-Point (CP) may be defined in the geometric center of the wheelbase. Each wheel (and each pivot) is *h* distance from the chassis CP, where:

$$h = \frac{\sqrt{l^2 + w^2}}{2} \qquad \text{eq. 1}$$

Direction of Travel is not necessarily aligned with the wheelbase. The difference between the chassis orientation and the direction of travel is γ. It is important than γ is known. Right-hand-rule applies to angle measurements with the thumb on the z (up) axis. All angle calculations will be carried out in radians. All angular values expressed here will be in radians unless otherwise noted.

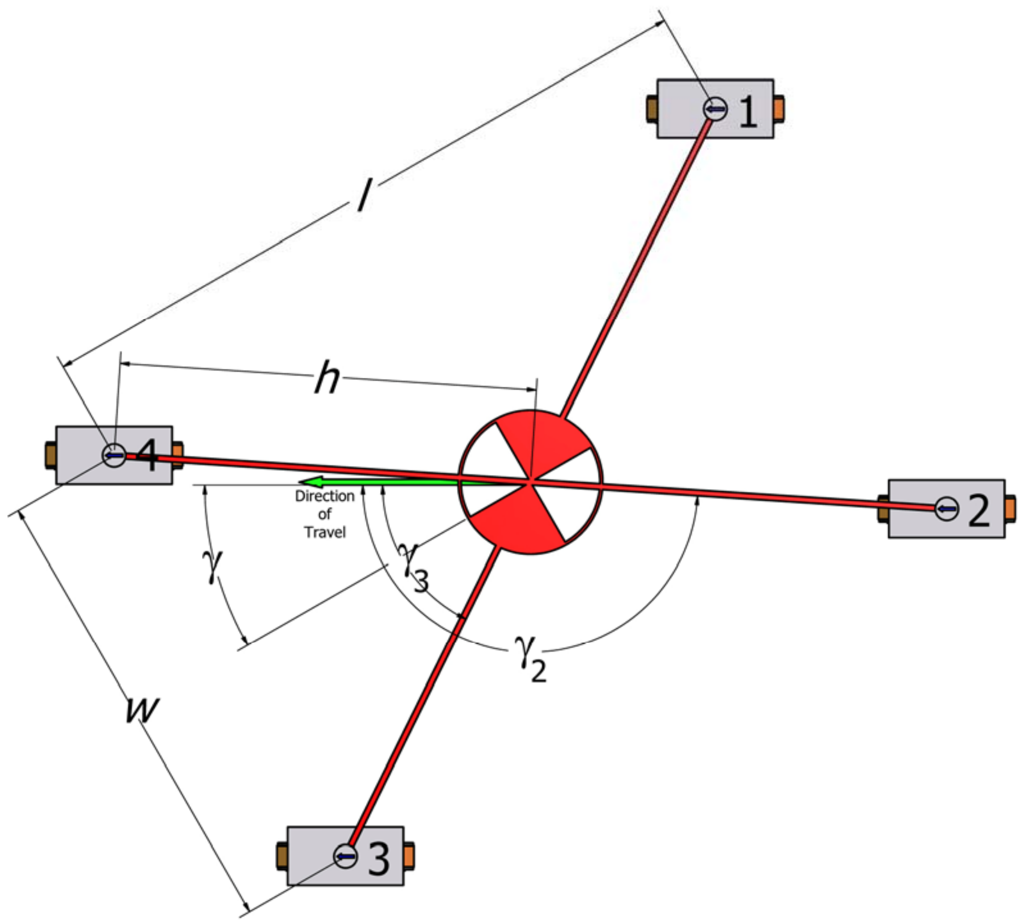
Normally in Crab Mode, all wheels will be aligned with the direction of travel and all drives will be equally powered (or, ideally, all driven at the same speed, which is not necessarily the same thing). All wheels would therefore be pivoted at

$$\alpha = 2\pi - \gamma \qquad \text{eq. 2}$$

from "straight ahead".

The orientation of each pivot/wheel from the chassis CP may also be defined relative to the direction of travel, γ_i (i = 1-4).

FIGURE I – Basic Crab Chassis



Crab Modes

There are two logical ways to control Crab Mode. There are also two logical ways to twist in Crab Mode. Each of the two twist modes fits intuitively (also logically and philosophically) with its own Crab Mode. We will describe the two Crab Modes before describing the Twist Modes.

Crab Mode 1 – Anchored to the chassis

In this mode, the direction of the joystick's deviation from neutral corresponds directly to the chassis's direction of travel, relative to the chassis's orientation. Pushing the joystick straight ahead moves the chassis straight forward ($\gamma = 0$). Throwing the joystick directly right moves the chassis towards the chassis's right side ($\gamma = 3\pi/2$). Pulling the joystick straight back moves the chassis backwards ($\gamma = \pi$). A good mode if chassis orientation means something.

It is not clear that the drive motors would ever reverse in Crab 1.

Crab Mode 2 – Free form *(but I know where I am, really!)*

I am certain this is how Team 118 worked in 2007 (except maybe the knowing part).

In Crab 2, y-direction joystick movement runs the drive motors (forward or reverse). x-direction joystick movement runs the steering motors (left and right). A neutral x-position means that the robot continues on its current heading regardless of that heading's orientation with the chassis.

x-direction joystick movement changes heading, without regard to chassis orientation.

There is a presumption here that one wheel will be the master and the other three slaves. One wheel must set the heading. The other three need to point in the same direction.

This could be very intuitive for a driver (looking for movement without having to pay and attention to chassis orientation). Visually, we are directly attuned to movement. Attention to orientation requires thought, slowing the process.

This is as far as I care to judge Crab 1 & 2 without data. They are different, though. Fortunately, through the magic of programming, we can (in principle) accommodate both.

Twist Modes

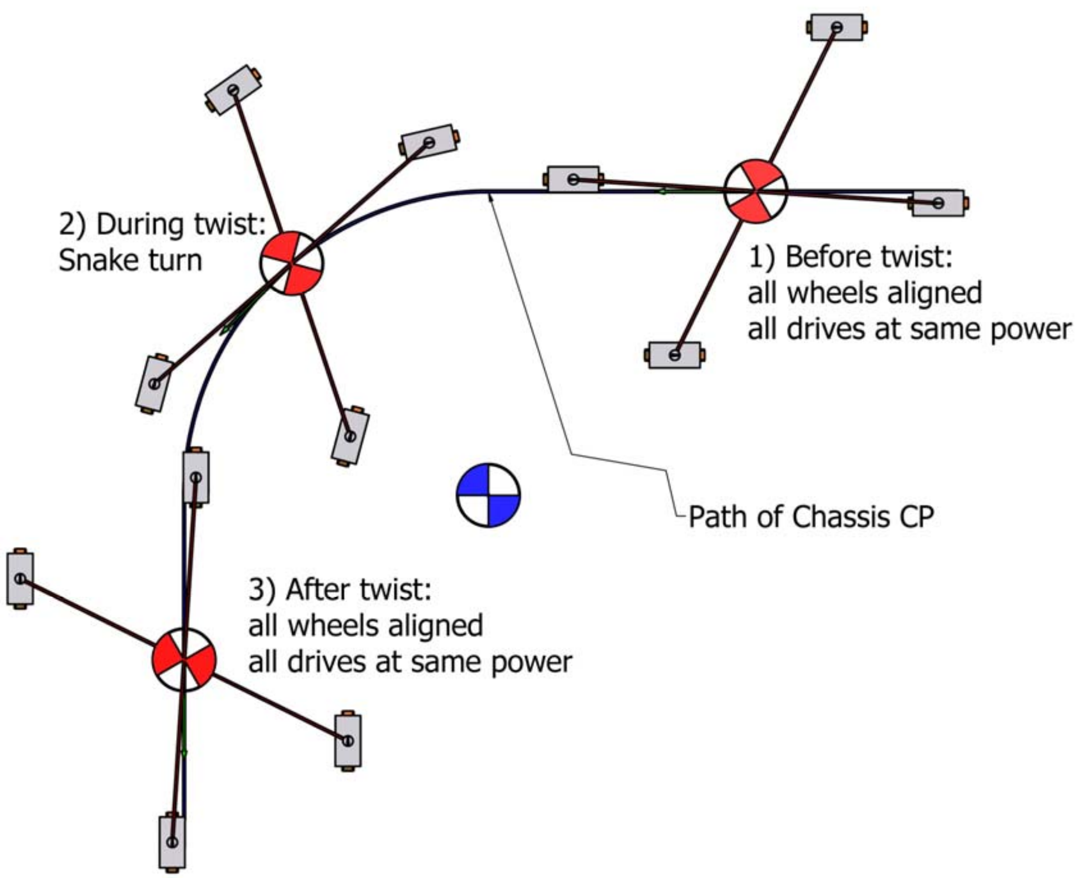
Twist 1

Crab 1 locks joystick orientation with chassis drive direction. Twist 1 should be consistent. This can be accomplished by incorporating a Snake Mode turn into a Crab Mode 1 Twist. Such a turn is shown schematically in Figure 2.

Through the turn, the orientation of the chassis relative to the direction of travel (γ) never changes. Chassis orientation does change relative to the field, and does so in an overt, controlled manner.

Twist 1 is really a *turn*, not a *twist*, but it is a Snake turn performed in Crab Mode.

FIGURE 2 – Twist 1 Schematic

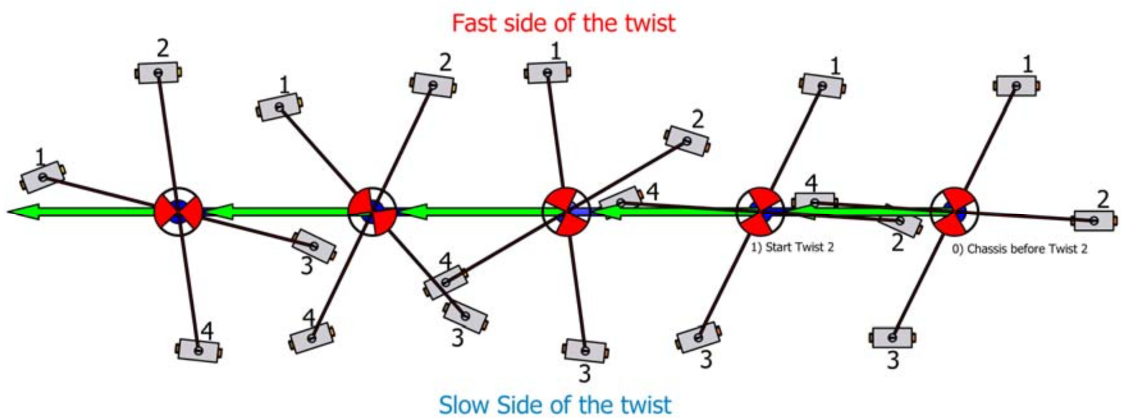


<u>Twist 2</u>

In Crab 2, steering is relative to the chassis's current heading in relation to the field. There is no fixed relationship between joystick position and chassis heading (γ). In Crab 2, it would make sense for Twist to rotate the chassis without (if possible) changing the heading relative to the field. This would necessarily change γ.

So Twist 2 is a real *twist* and not a *turn*. Twist 2 is shown schematically in Figure 3.

FIGURE 3 – Twist 2 Schematic



Twist 2 introduces a new steering regime to 1640 – dynamic steering.

Up to this point (including Twist 1), steering can be considered static. That is, once the human controls are set, this determines a robot response (steering angle & drive speed, in this case) which does not change until the human controls change.

Twist 2 is different. In order to rotate the chassis while retaining a constant bearing (relative to the field), steering and drive speed need to be dynamic. Steering angle and drive speed changes continuously for each wheel as a function of γ_i (for given human interface values). This will provide a unique challenge for the team.

Twist 3

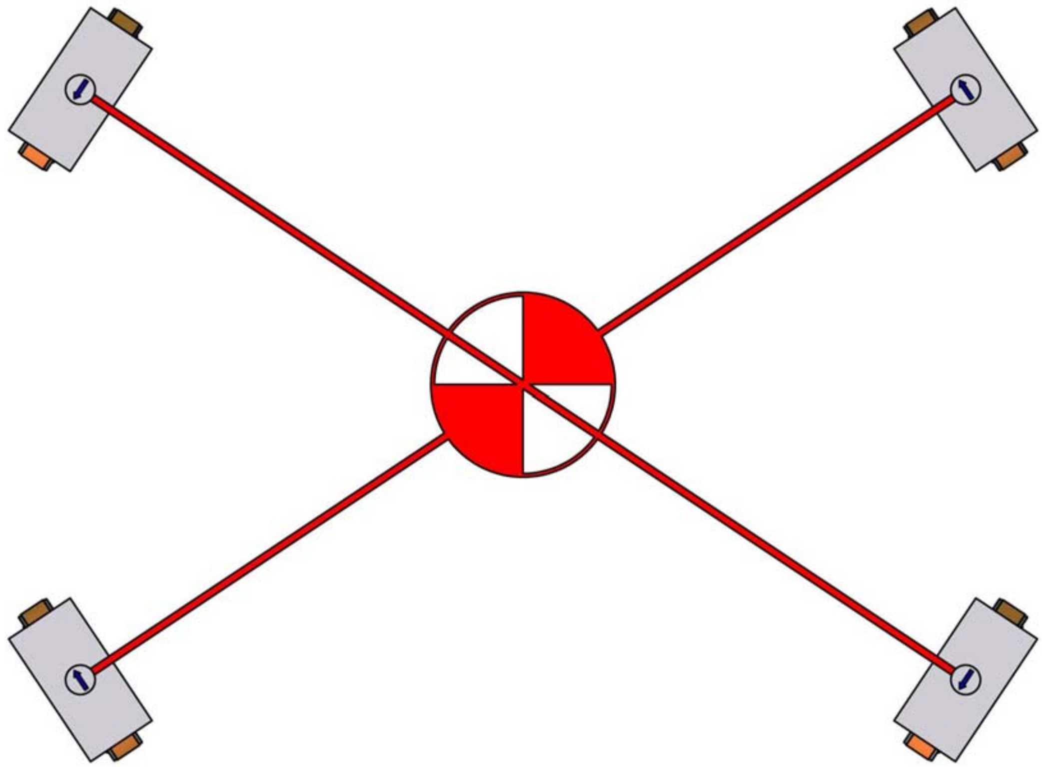
Did I say 2 twist modes? What was I thinking? There's a caveat.

If the robot is stationary, a 3rd twist is logical. This simply sets all of the wheels tangent to the circle that they describe and turn the chassis like a turret. Ken Au has already programmed this as an independent mode. This is also the logical limit of a Snake Mode turn (but in Snake the "inside" pivots turn an extra π). See Figure 4.

In the configuration shown, it would be necessary to run two motors in reverse to rotate the chassis.

We will see that Twist 3 is logically incorporated in Twist 2 without a special effort.

FIGURE 4 – Twist 3 (Stationary Chassis Rotation)



The Math behind Twist 1

Twist 1 is a generalized approach to Snake Mode.

Figure 5 defines the basic geometry behind Twist 1. The chassis is represented by a circle of radius *h* about a chassis CP. There is an angular bearing of travel relative to the nominal chassis orientation of γ. There are *n* (*n* = 4 in our case, but this is not important here) wheels distributed around the circle's radius (only one is shown), each at a known, fixed orientation ($\gamma_i°$) relative to the physical chassis, but at a variable orientation relative to the direction of travel (γ_i), so that:

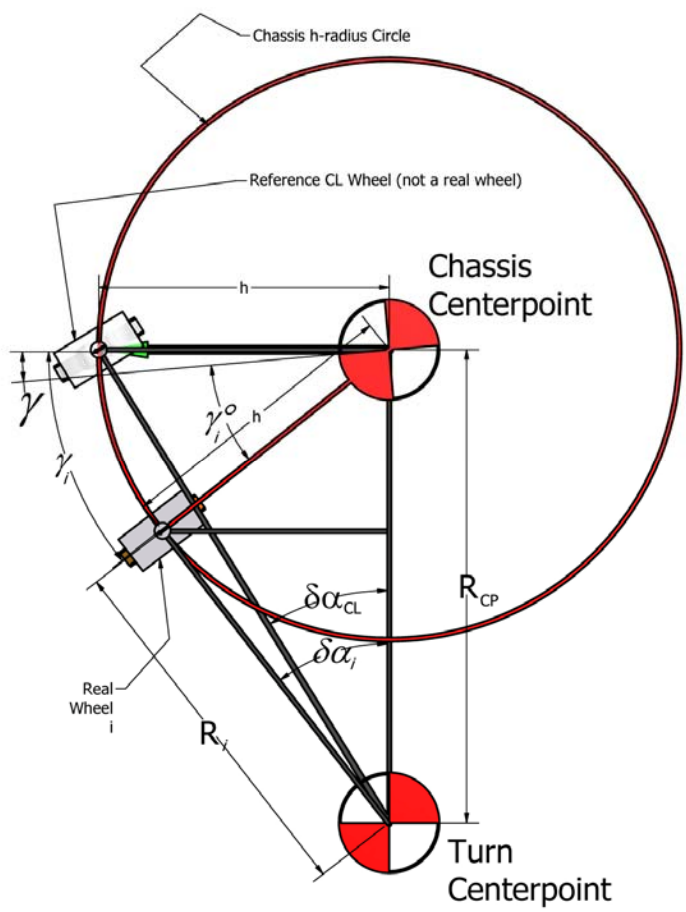
$$\gamma_i = \gamma + \gamma_i^{\circ} \qquad \text{eq. 3}$$

Care needs to be taken to check that γ_i remains in the range 0 - 2π (add or subtract 2π to if necessary).

When driving in Crab Mode (without twist), all wheels are oriented at $\alpha = 2\pi - \gamma$ relative to the chassis's "straight ahead" orientation.

As with the earlier Snake Mode analysis, it is useful to imagine a "reference wheel" on the centerline relative to the direction of travel. This reference wheel is located h distance ahead of the chassis CP. The centerline reference wheel would respond directly and proportionally to the joystick z-axis input and therefore determine the turn radius (R_{CL}) as a function of reference pivot angle ($\delta\alpha_{CL}$). The nomenclature $\delta\alpha$ is adopted here because it is an additional steering change on top of the Crab steering angle α.

FIGURE 5 – Twist 1 Geometry



$\delta\alpha_{CL}$ is assigned proportionally from the joystick z-axis input. For the sake of simplicity, I limited the maximum $\delta\alpha_{CL}$ values to the range ±π/4. This limitation keeps the turning centerpoint from coming inside the chassis circle. In Snake Mode, the limitation is ±π/2. The chassis turn radius R_{CP} is calculated:

$$R_{CP} = \frac{h}{\tan \alpha_{CL}} \qquad \text{eq. 4}$$

A turn radius for each wheel (R_i) can be calculated.

$$R_i = \sqrt{h^2 \cos^2 \gamma_i + \left(R_{CP} - h \sin \gamma_i\right)^2} \qquad \text{eq. 5}$$

As well as a twist steering angle ($\delta\alpha_i$). *(Note: the following equation may need to be negated or otherwise modified based on joystick input specs).*

$$\delta\alpha_i = [joystick\_z\_sign] \times \sin^{-1}\left(\frac{h \cos \gamma_i}{R_i}\right) \qquad \text{eq. 6}$$

The wheel drive velocity factor for each wheel is the ratio:

$$v_i = \frac{R_i}{R_{\max}} \qquad \text{eq. 7}$$

where R_{max} is the greatest turning radius for the wheels.

A worksheet for Twist 1 calculations was developed.

Twist 1 Pivot Model

l	**31**	in
w	**21**	in
h	**18.72**	in

Yellow Field - Model Input
BOLD - System Constants

			1	2	3	4
		γ_i°	**3.737**	**2.546**	**0.595**	**5.688**
γ	0.524	γ_i	4.261	3.070	1.119	6.211
		h sin γ_i	-16.84	1.34	16.84	-1.34
		h cos γ_i	-8.17	-18.67	8.17	18.67

z	α_{CL}	R_{CP} in	R_i (in) 1	2	3	4	R_{max} in	$\delta\alpha_i$ 1	2	3	4	v_i 1	2	3	4
-1.0	-0.785	-18.7	8.4	27.4	36.5	25.5	36.5	1.345	0.749	-0.226	-0.821	0.23	0.75	1.00	0.70
-0.9	-0.707	-21.9	9.6	29.8	39.6	27.8	39.6	1.015	0.676	-0.208	-0.737	0.24	0.75	1.00	0.70
-0.8	-0.628	-25.8	12.1	32.9	43.4	30.7	43.4	0.741	0.603	-0.190	-0.653	0.28	0.76	1.00	0.71
-0.7	-0.550	-30.6	16.0	37.0	48.1	34.7	48.1	0.538	0.530	-0.171	-0.569	0.33	0.77	1.00	0.72
-0.6	-0.471	-36.7	21.5	42.4	54.2	40.0	54.2	0.390	0.456	-0.151	-0.485	0.40	0.78	1.00	0.74
-0.5	-0.393	-45.2	29.5	50.1	62.6	47.7	62.6	0.281	0.382	-0.131	-0.403	0.47	0.80	1.00	0.76
-0.4	-0.314	-57.6	41.6	61.8	74.9	59.3	74.9	0.198	0.307	-0.109	-0.320	0.56	0.83	1.00	0.79
-0.3	-0.236	-78.0	61.7	81.5	95.2	78.9	95.2	0.133	0.231	-0.086	-0.239	0.65	0.86	1.00	0.83
-0.2	-0.157	-118.2	101.7	121.0	135.3	118.3	135.3	0.080	0.155	-0.060	-0.158	0.75	0.89	1.00	0.87
-0.1	-0.079	-237.9	221.2	240.0	254.9	237.3	254.9	0.037	0.078	-0.032	-0.079	0.87	0.94	1.00	0.93
0.0	0.000														
0.1	0.079	237.9	254.9	237.3	221.2	240.0	254.9	-0.032	-0.079	0.037	0.078	1.00	0.93	0.87	0.94
0.2	0.157	118.2	135.3	118.3	101.7	121.0	135.3	-0.060	-0.158	0.080	0.155	1.00	0.87	0.75	0.89
0.3	0.236	78.0	95.2	78.9	61.7	81.5	95.2	-0.086	-0.239	0.133	0.231	1.00	0.83	0.65	0.86
0.4	0.314	57.6	74.9	59.3	41.6	61.8	74.9	-0.109	-0.320	0.198	0.307	1.00	0.79	0.56	0.83
0.5	0.393	45.2	62.6	47.7	29.5	50.1	62.6	-0.131	-0.403	0.281	0.382	1.00	0.76	0.47	0.80
0.6	0.471	36.7	54.2	40.0	21.5	42.4	54.2	-0.151	-0.485	0.390	0.456	1.00	0.74	0.40	0.78
0.7	0.550	30.6	48.1	34.7	16.0	37.0	48.1	-0.171	-0.569	0.538	0.530	1.00	0.72	0.33	0.77
0.8	0.628	25.8	43.4	30.7	12.1	32.9	43.4	-0.190	-0.653	0.741	0.603	1.00	0.71	0.28	0.76
0.9	0.707	21.9	39.6	27.8	9.6	29.8	39.6	-0.208	-0.737	1.015	0.676	1.00	0.70	0.24	0.75
1.0	0.785	18.7	36.5	25.5	8.4	27.4	36.5	-0.226	-0.821	1.345	0.749	1.00	0.70	0.23	0.75

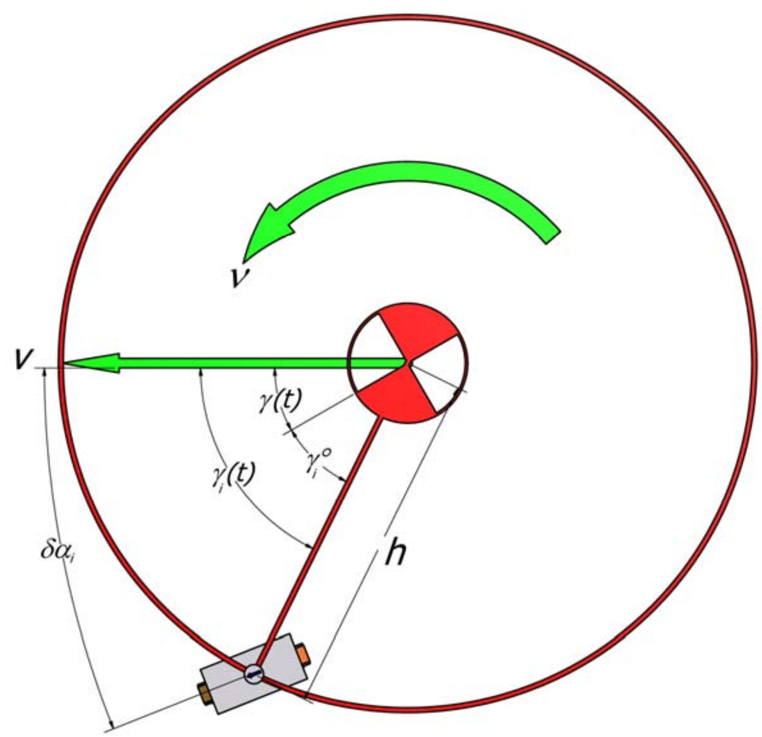
The most significant complication of Twist 1 as compared to Snake Mode is that the calculations depend upon the chassis orientation γ, and can therefore not be easily pre-calculated (unless you parse γ and pre-calculate a 3-d array).

Twist 1 is a static steering system. Once the joystick controls are set, all steering and drive values are determined and do not change until the joystick controls change.

Math behind Twist 2

Twist 2 geometry is shown schematically in Figure 6. It's actually simpler than Twist 1. It retains the useful concept of the *h*-radius chassis circle. This chassis moves at a velocity V (in/s) and rotates, or twists, at a rate of ν (radians/s) *[Note: a capital V is used here for velocity to avoid confusion with ν (rotational rate) in equations]*. The imaginary Centerline Wheel concept used in Twist 1 is not useful for Twist 2. We need look at only one wheel, *i* (there are *n* wheels). The wheel has a fixed orientation relative to the chassis ($\gamma_i°$ – same as Twist 1). γ is the chassis orientation relative to the direction of travel and is now a function of *t* (time), where $\gamma(0)$ is a known initial condition. Steering twist angle ($\delta\alpha_i$) is the deviation from Crab Mode steering angle (α). $\alpha = 2\pi - \gamma$ is of course also a function of *t* ($\alpha(t)$).

FIGURE 6 – Twist 2 Geometry



Twist 2 rotational rate ν competes with chassis velocity V. Any wheel passing $\gamma_i = 3\pi/2$ with a positive ν (or $\gamma_i = \pi/2$ with a negative ν) must be driven at a speed of $V + h|\nu|$. Since the maximum drive speed is limited to V_{max} (about 108 in/s), chassis and rotational velocities will need to be balanced on the basis of y & z-axis Joystick inputs.

The attached Model contains a reasonable allocation between chassis and rotational velocities. At zero forward velocity and maximum z, the chassis will rotate at full drive speed. At zero twist (z) and maximum drive (y), the chassis will drive forward at full drive speed. In the arena of negotiation, drive speed remains maximized at its peek and the twist and forward drive play off against each other.

The rotational position of the chassis can calculated:

$$\gamma(t) = \gamma(0) + \nu t \qquad \text{eq. 8}$$

It is important to keep the value of γ between 0 and 2π.

To calculate the angular positions any for individual wheel i:

$$\gamma_i(t) = \gamma + \gamma_i^{\circ} \qquad \text{eq. 9}$$

As above, keep these values between 0 and 2π.

x, y and scalar wheel velocities are:

$$V_{x,i}(t) = V - h\nu \sin\gamma_i \qquad \text{eq. 10}$$

$$V_{y,i}(t) = h\nu \cos\gamma_i \qquad \text{eq. 11}$$

$$V_i(t) = \sqrt{V_{x,i}^2 + V_{y,i}^2} \qquad \text{eq. 12}$$

The calculation of $\delta\alpha_i(t)$ is conditional.

If $V_{x,i} > 0$, then:

$$\delta\alpha_i = \tan^{-1}\frac{V_{y,i}}{V_{x,i}} \qquad \text{eq. 13a}$$

If $V_{x,i} > 0$ and $\gamma_i < \pi/2$, then:

$$\delta\alpha_i = \tan^{-1}\frac{V_{y,i}}{V_{x,i}} + \pi \qquad \text{eq. 13b}$$

If $V_{x,i} > 0$ and $\gamma_i \geq \pi/2$, then:

$$\delta\alpha_i = \tan^{-1}\frac{V_{y,i}}{V_{x,i}} - \pi \qquad \text{eq. 13c}$$

There is a dimensionless number, $h|\nu|/V$, which characterizes the behavior of Twist 2. If $h|\nu|/V < 1$, the wheels do not pivot completely around with each twist revolution (they oscillate). If, on the other hand, $h|\nu|/V \geq 1$, then the wheels do pivot completely around with each twist revolution.

These equations will work if $V = 0$. In this case, Twist 2 becomes effectively Twist 3.

A robust model has been developed based on 1st principles. An earlier model had been built from the back-end forward, but it broke under the conditions $h|\nu|/V \geq 1$. Under the conditions $h|\nu|/V < 1$, the 1st principles and back-end models agree exactly. The solution for the new model is analytical. The worksheet model is shown below.

Twist 2 Pivot Model

l	**31**	in	x			V	75.6	in/s
w	**21**	in	y	0.7		v	-1.731	s^{-1}
h	**18.72**	in	z	-0.3		hv/V	0.429	
Δt	0.1	s	V_{max}	108	in/s			

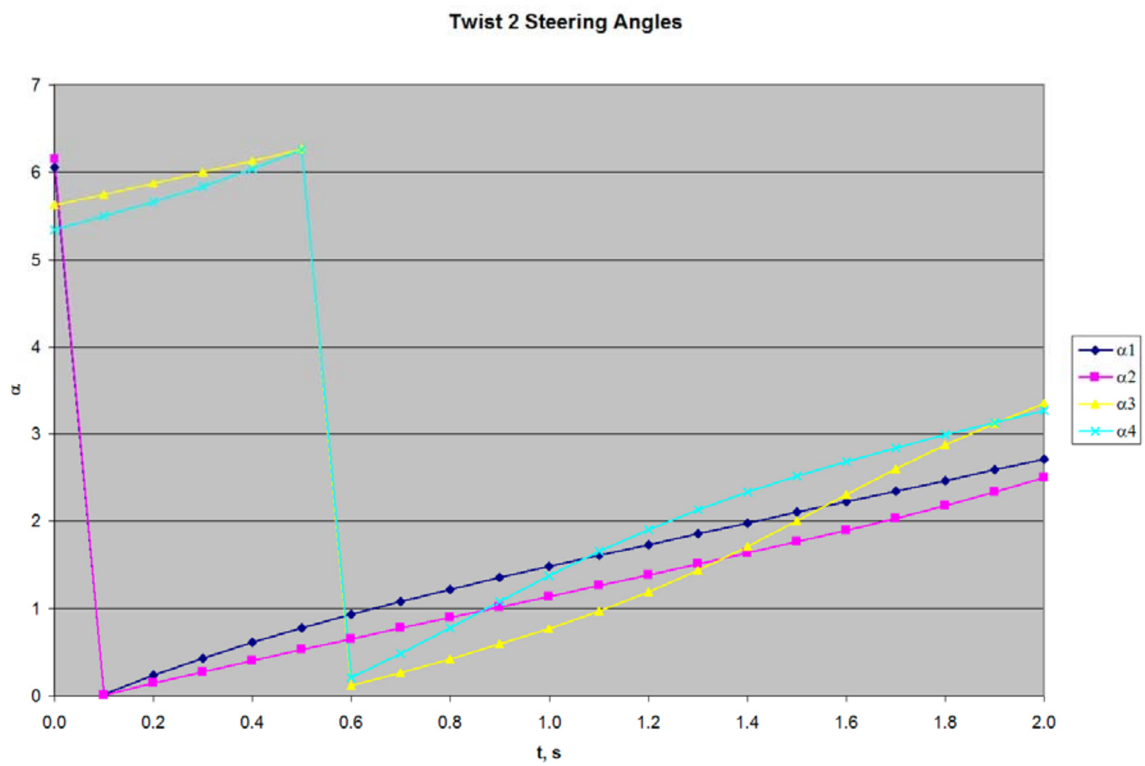
Yellow Field - Model Input

BOLD - System Constants

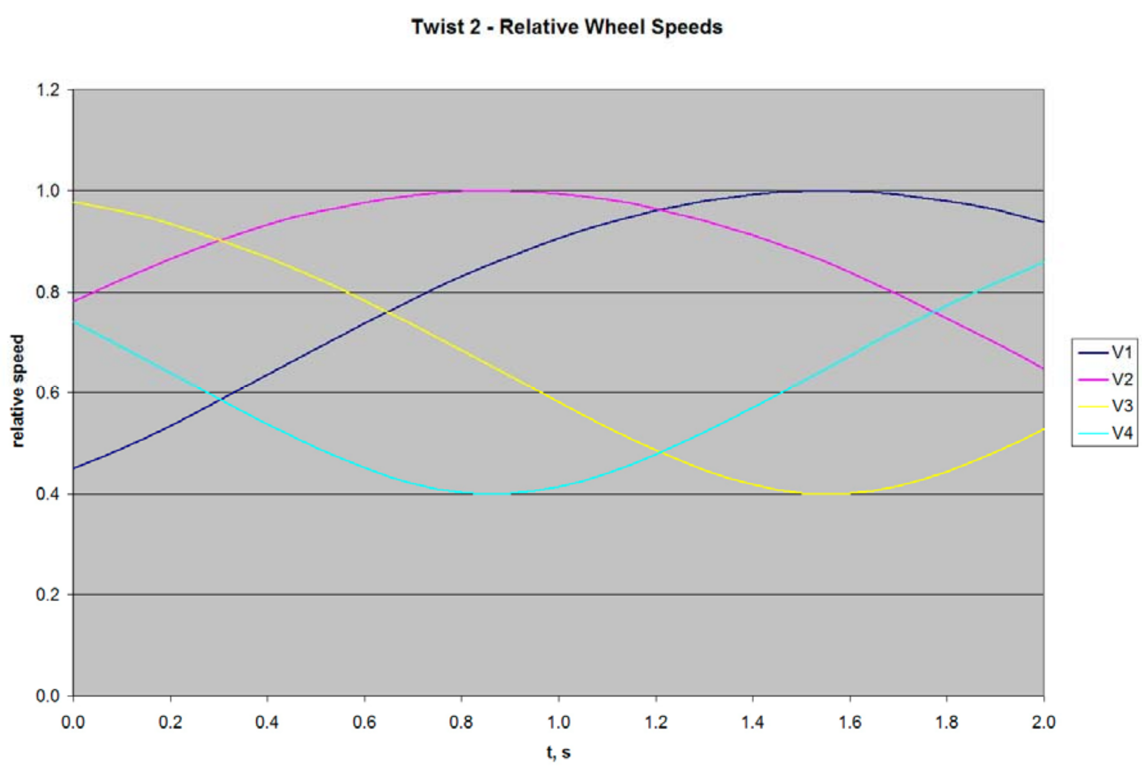
$\gamma(0)$	0.524	$\gamma_i°$	**3.737**	**2.546**	**0.595**	**5.688**

t	γ	α	$V_{x,i}$				$V_{y,i}$				$V_{i\,(relative)}$				$\delta\alpha_i$				α_i			
			1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
0.00	0.52	5.76	46.5	77.9	104.7	73.3	14.1	32.3	-14.1	-32.3	0.45	0.78	0.98	0.74	0.30	0.39	-0.13	-0.42	6.06	6.15	5.63	5.34
0.10	0.35	5.93	49.3	83.5	101.9	67.7	19.0	31.4	-19.0	-31.4	0.49	0.83	0.96	0.69	0.37	0.36	-0.18	-0.43	0.02	0.01	5.75	5.50
0.20	0.18	6.11	53.0	88.8	98.2	62.4	23.2	29.6	-23.2	-29.6	0.54	0.87	0.93	0.64	0.41	0.32	-0.23	-0.44	0.24	0.14	5.87	5.66
0.30	0.00	6.28	57.3	93.7	93.9	57.5	26.7	26.9	-26.7	-26.9	0.59	0.90	0.90	0.59	0.44	0.28	-0.28	-0.44	0.43	0.28	6.00	5.84
0.40	6.11	0.17	62.2	98.0	89.0	53.2	29.5	23.4	-29.5	-23.4	0.64	0.93	0.87	0.54	0.44	0.23	-0.32	-0.41	0.61	0.40	6.13	6.04
0.50	5.94	0.34	67.5	101.7	83.7	49.5	31.4	19.2	-31.4	-19.2	0.69	0.96	0.83	0.49	0.44	0.19	-0.36	-0.37	0.78	0.53	6.27	6.26
0.60	5.77	0.51	73.0	104.6	78.2	46.6	32.3	14.4	-32.3	-14.4	0.74	0.98	0.78	0.45	0.42	0.14	-0.39	-0.30	0.93	0.65	0.12	0.21
0.70	5.60	0.69	78.6	106.7	72.6	44.5	32.3	9.2	-32.3	-9.2	0.79	0.99	0.74	0.42	0.39	0.09	-0.42	-0.20	1.08	0.77	0.27	0.48
0.80	5.42	0.86	84.1	107.8	67.1	43.4	31.3	3.7	-31.3	-3.7	0.83	1.00	0.69	0.40	0.36	0.03	-0.44	-0.09	1.22	0.90	0.42	0.78
0.90	5.25	1.03	89.4	107.9	61.8	43.3	29.3	-1.9	-29.3	1.9	0.87	1.00	0.63	0.40	0.32	-0.02	-0.44	0.04	1.35	1.02	0.59	1.08
1.00	5.08	1.21	94.2	107.1	57.0	44.1	26.5	-7.4	-26.5	7.4	0.91	0.99	0.58	0.41	0.27	-0.07	-0.44	0.17	1.48	1.14	0.77	1.37
1.10	4.90	1.38	98.5	105.4	52.7	45.8	22.9	-12.8	-22.9	12.8	0.94	0.98	0.53	0.44	0.23	-0.12	-0.41	0.27	1.61	1.26	0.97	1.65
1.20	4.73	1.55	102.1	102.7	49.1	48.5	18.6	-17.7	-18.6	17.7	0.96	0.97	0.49	0.48	0.18	-0.17	-0.36	0.35	1.73	1.38	1.19	1.90
1.30	4.56	1.73	104.9	99.3	46.3	51.9	13.8	-22.1	-13.8	22.1	0.98	0.94	0.45	0.52	0.13	-0.22	-0.29	0.40	1.86	1.51	1.44	2.13
1.40	4.38	1.90	106.9	95.1	44.3	56.1	8.5	-25.9	-8.5	25.9	0.99	0.91	0.42	0.57	0.08	-0.27	-0.19	0.43	1.98	1.63	1.71	2.33
1.50	4.21	2.07	107.9	90.4	43.3	60.8	3.0	-28.8	-3.0	28.8	1.00	0.88	0.40	0.62	0.03	-0.31	-0.07	0.44	2.10	1.76	2.00	2.52
1.60	4.04	2.25	107.9	85.2	43.3	66.0	-2.6	-30.9	2.6	30.9	1.00	0.84	0.40	0.67	-0.02	-0.35	0.06	0.44	2.22	1.90	2.30	2.68
1.70	3.86	2.42	107.0	79.7	44.2	71.5	-8.1	-32.1	8.1	32.1	0.99	0.80	0.42	0.73	-0.08	-0.38	0.18	0.42	2.34	2.04	2.60	2.84
1.80	3.69	2.59	105.1	74.1	46.1	77.1	-13.4	-32.4	13.4	32.4	0.98	0.75	0.44	0.77	-0.13	-0.41	0.28	0.40	2.47	2.18	2.87	2.99
1.90	3.52	2.76	102.4	68.6	48.8	82.6	-18.3	-31.6	18.3	31.6	0.96	0.70	0.48	0.82	-0.18	-0.43	0.36	0.37	2.59	2.33	3.12	3.13
2.00	3.35	2.94	98.8	63.2	52.4	88.0	-22.6	-29.9	22.6	29.9	0.94	0.65	0.53	0.86	-0.22	-0.44	0.41	0.33	2.71	2.50	3.34	3.27

This is a dynamic system. Setting the joystick position simply starts the dynamic process. Steering angles for the above twist are:



And relative motor speeds:



But each system of inputs provides a unique solution.