Pivot-Wheel Drive

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4-Wheel Independent Pivot-Wheel Drive describes a 4wd drive-train in which each of the (4) wheels are independently driven and may be independently pivoted for steering purposes. The design offers the potential for excellent drive-train performance and a solution to conventional (tank) drive-train design constraints. The design also brings some clear design and control challenges.

This arrangement provides the possibility to operate in several different modes:

1. Crab mode – pivots all wheels together and at common speed to steer the robot in any direction on the 2-d playing surface (true 2-d drive). The mode does not control chassis orientation.
2. Snake mode – pivots front and rear wheels in opposite directions to guide the robot through a turn
    a. X-bias – *drive direction aligned with the long-axis of the robot*
    b. Y-bias – *drive direction aligned with the short-axis of the robot*
3. Automobile mode – pivots front wheels only to guide robot through a turn
    a. X-bias
    b. Y-bias
4. Tank mode – Does not use pivots to steer (but can use pivots to change drive orientation *ala Twitch*). Steering accomplished by differential L & R drive speeds.
    a. X-bias
    b. Y-bias

Mechanical and Control Considerations

Both mechanical and control requirements vary between these four (or seven) modes, with Crab being very different from all of the rest. We'll therefore deal with this first.

Crab Mode

Crab pivots all (4) wheels in unison and drives all at the same speed. For Crab to work properly, the pivoting needs to be limitless (no stops), synchronous (all wheels kept in alignment) and wheels need to be driven at substantially the same speed. There is no need for reverse (forward only). A single joystick controls speed (displacement from center) and direction (x-y direction of displacement).

Strength is true 2-d maneuverability

Weakness is no overt control over chassis orientation.

Team 118's very capable 2007 robot[i] utilized a single drive system and a common steering drive, both linked via chain drive to the four wheels. This kept the wheels pivoted in the same direction and moving the same speed mechanically. Drive power ran through the pivots coaxially, so pivoting was unlimited. It's not clear that 118 needed to measure the pivot angle. Chassis orientation was uncontrollable (although the cited reference contradicts this). 118's arm was on a turret, which provided 360° play so this was not a problem from the standpoint of hanging ringers. I imagine it did give the robot problems with climbing ramps for bonus points.

Crab mode makes two unique design constraints:

1. Drive power needs to be transmitted to the wheel coaxially through the pivot
2. If wheels are synchronized by the control system, pivot angle needs to be measured using an unlimited-turn device (such as an encoder) with a reference (to calibrate – assume once per rev).

Snake mode

Snake mode also pivots all four wheels to steer. In Snake, wheels pivot to the turn tangents, thereby largely eliminating the frictional resistance against turning characteristic of Tank drive.

To do snake right:

- Rear wheels pivot same angle but opposite direction as front
- Inside-of-turn wheels should pivot more than outside-of-turn (because turn radius is smaller).
- Inside-of-turn wheels should run at lower speed than outside-of-turn wheels (because turn circumference is smaller) and this reduction is a function of degree of turn.
- The logical limit of snake turning is for the robot to spin around its centerpoint.
- Snake wheels need pivot 180° (to go from spinning around CP one direction to spinning around CP in the opposite).
- Single joystick arcade control makes sense. x-motion drives pivot and speed differential; y-motion drives overall speed. Reverse is useful.

The Mathematics:

Coordinates are set using right hand rule. The x-axis points straight out of the robot's front; y-axis out of the robot's left side; z-axis straight up. Pivoting the front wheels in a positive angle turns the robot left.

Wheelbase length (*l*) and width (*w*) need to be defined. I used inches.

To clarify the inside-outside wheel effects, I found it useful to imagine a set of centerline wheels. The centerline wheels would respond directly to the joystick and determined the turn radius (R_{CL}) as a function of pivot angle (α_{CL}).

$$R_{CL} = \frac{l}{2\sin\alpha_{CL}} \qquad \text{eq 1}$$

α_{CL} can range from 0 to 90° (or –90° to 90°).

It's also useful to define the robot center-point (the geometric center of the drive-train) and the radius from the turn center to the robot center-point (R_{CP}).

$$R_{CP} = \sqrt{R_{CL}^2 - \frac{l^2}{4}} = \frac{l}{2}\sqrt{\frac{1}{\sin^2\alpha_{CL}} - 1} = \frac{l/2}{\tan\alpha_{CL}} \qquad \text{eq 2}$$

Note that for a given steering angle (α_{CL}), turn radius (R_{CP}) is proportional to drive-train length (*l*). With R_{CP} defined, the inside and outside pivot angles (α_i & α_o, respectively) can be calculated.

For α_o:

$$\alpha_o = \tan^{-1}\left(\frac{l/2}{R_{CP} + w/2}\right) = \tan^{-1}\left(\frac{1}{\frac{1}{\tan\alpha_{CL}} + \frac{w}{l}}\right) \qquad \text{eq 3}$$

For α_i, if $R_{CP} > w/2$ (if the center of the turn is outside the wheelbase):

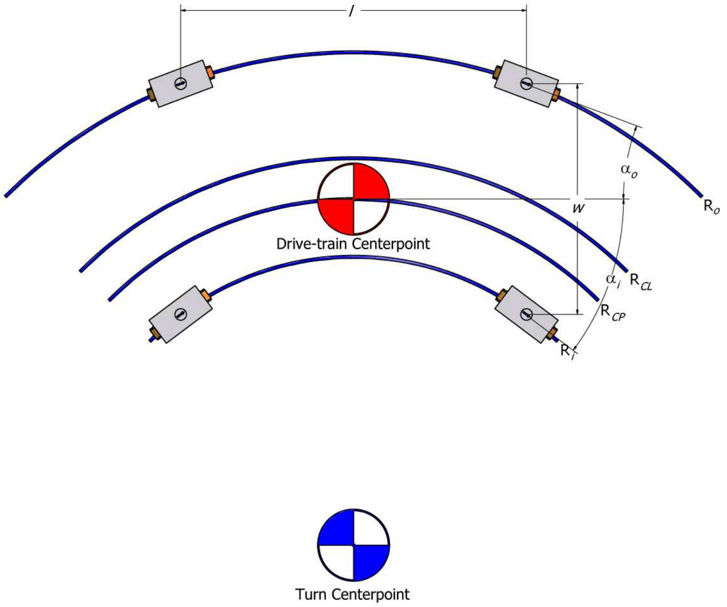
$$\alpha_i = \tan^{-1}\left(\frac{l/2}{R_{CP} - w/2}\right) \qquad \text{eq 4}$$

For α_i, if $R_{CP} < w/2$ (if the center of the turn is inside the wheelbase):

$$\alpha_i = \pi - \tan^{-1}\left(\frac{l/2}{R_{CP} - w/2}\right) \qquad \text{eq 5}$$

If $R_{CP} = w/2$ (where the turn center is on the wheelbase line and the above equations fall apart), $\alpha_i = \pi/2$. Note that angles are calculated in radians (and converted later to degrees for convenience).

A schematic view of the system is shown below:



The final important value is the inside wheel speed reduction. This is equal to the inside:outside turn circumference ratio, which in turn is equal to the inside:outside turn radius ratio (R_i/R_o). Both R_i and R_o can be calculated using equation 1 (above), by substituting α_i or α_o for α_{CL} in that equation.

A worksheet for Snake drive calculations was developed. For a 30" long x 20" drive-train, the results are:

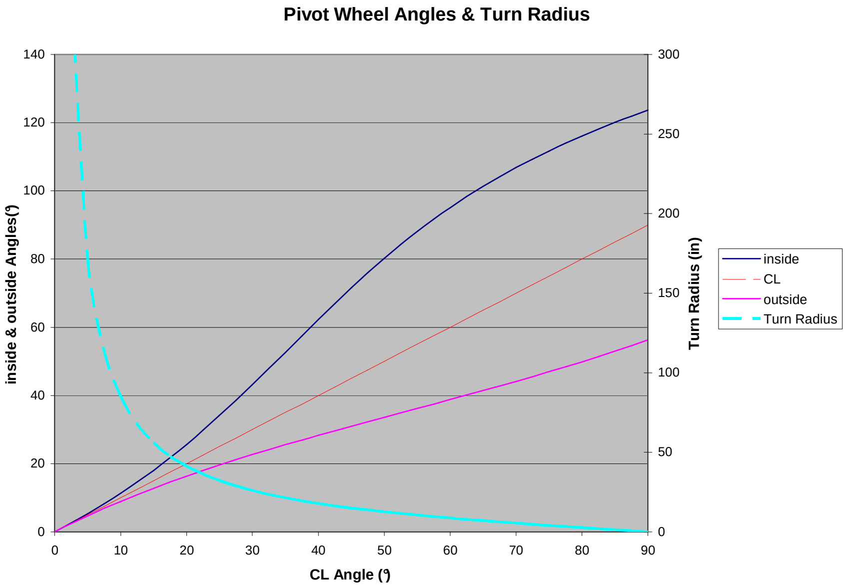
Pivot Wheel Robot - Snake Drive Calculations

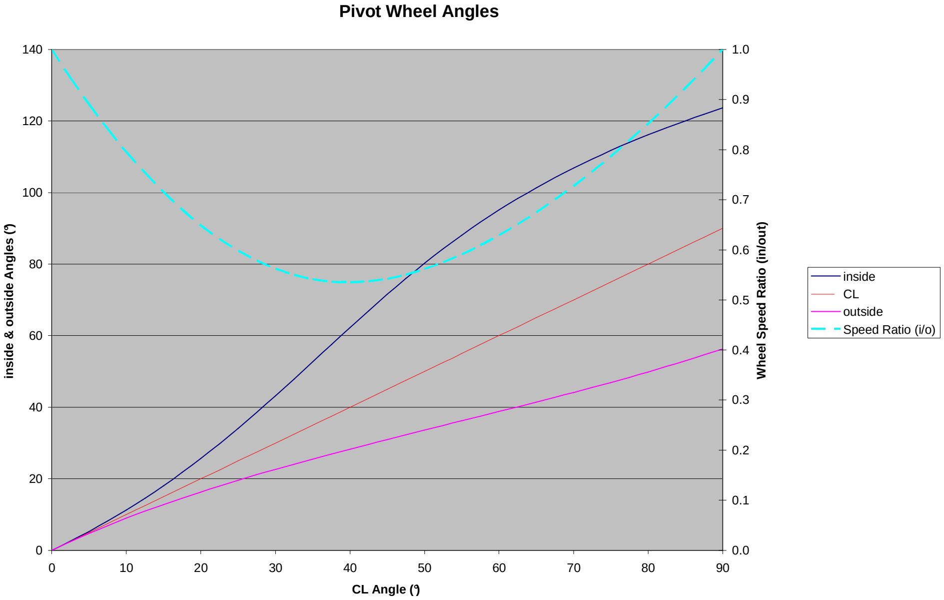
Robot Parameters

l	30	in
w	20	in

Turning angle (°)			Turn Radius (inches)				Speed Ratio
inside	CL	outside	R_{CP}	R_i	R_{CL}	R_o	R_i/R_o
0.0	0.0	0.0	∞	∞	∞	∞	1.000
11.3	10.0	9.0	85.1	76.6	86.4	96.2	0.795
25.7	20.0	16.3	41.2	34.6	43.9	53.4	0.649
43.2	30.0	22.6	26.0	21.9	30.0	39.0	0.562
62.3	40.0	28.3	17.9	16.9	23.3	31.7	0.535
80.2	50.0	33.6	12.6	15.2	19.6	27.1	0.561
95.1	60.0	38.8	8.7	15.1	17.3	23.9	0.629
106.8	70.0	44.1	5.5	15.7	16.0	21.5	0.728
116.1	80.0	49.9	2.6	16.7	15.2	19.6	0.852
123.7	90.0	56.3	0.0	18.0	15.0	18.0	1.000

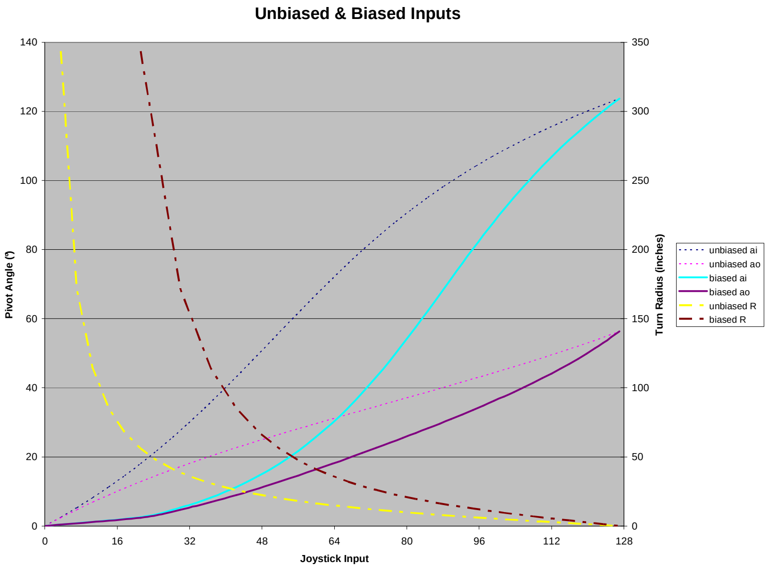
Graphically:





It ought to be possible to linearize the Pivot Angles.

Of greater concern is that as shown, the steering response is very sensitive at small angles. It becomes insensitive at large angles. It will probably be necessary to bias this in order to provide useful steering across the range. One such bias approach (square root) is shown below:



From a programming standpoint, such a bias could be managed using a lookup table.

Strengths:

- Efficient, hysteresis-free steering
- Responsive & intuitive
- High turning torque available
- Allows zero-radius turning in place
- May be switched between x and y bias

Weaknesses:

- Not a 2d drive
- The programming in not straightforward – doing this right presents clear control challenges
- May be overly sensitive for fine control due to large angles covered

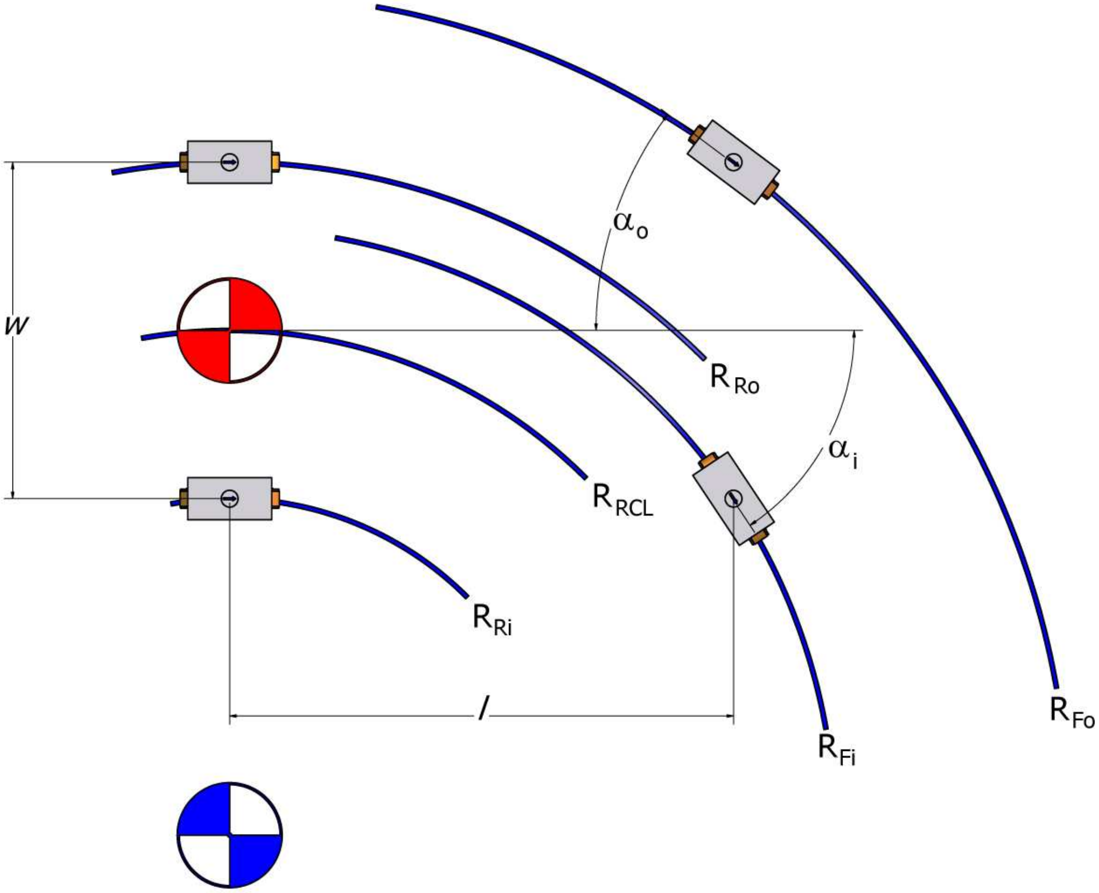
Automobile Mode

A limited version of Snake Mode. Front wheels only pivot. Pivot angle is much more limited (say 35-45°). Differential inside:outside pivot angles is a plus, but not essential (cars don't typically do this). Differential inside:outside drive speeds is important (cars do this via differential gearing), but will be more linear due to limited angle.

Mathematically, the Automobile drive functions (in terms of turning radius, α_i & α_o) like a Snake drive having a wheelbase twice as long. This means that for the same steering angle, the Automobile drive turning radius will be twice the Snake drive's. The larger turning radius combined with smaller steering angle range will provide Automobile drive with superior fine control.

The Mathematics:

See the schematic view, below:



Automobile's R_{RCL} is analogous to Snake's R_{CP}.

$$R_{RCL} = \frac{l}{\tan \alpha_{CL}} \qquad \text{eq 6}$$

Comparing with eq 2, this turning radius is twice Snake drive's.

Calculating outside and inside front pivot angles:

$$\alpha_o = \tan^{-1}\left(\frac{l}{R_{RCL} + {}^{w}\!/_{2}}\right) = \tan^{-1}\left(\frac{1}{\frac{1}{\tan\alpha_{CL}} + \frac{w}{2l}}\right) \qquad \text{eq 7}$$

Comparing eq 7 with eq 3 shows the doubling of effective wheelbase length between Snake and Automobile modes.

$$\alpha_i = \tan^{-1}\left(\frac{l}{R_{RCL} - {}^{w}\!/_{2}}\right) \qquad \text{eq 8}$$

Turning radii for individual wheels are:

$$R_{Ro} = R_{RCL} + {}^{w}\!/_{2} \qquad \text{eq 9}$$

$$R_{Ri} = R_{RCL} - {}^{w}\!/_{2} \qquad \text{eq 10}$$

$$R_{Fo} = \sqrt{R_{Ro}^2 + l^2} \qquad \text{eq 11}$$

$$R_{Fi} = \sqrt{R_{Ri}^2 + l^2} \qquad \text{eq 12}$$

Wheel speeds should be proportional to relative turning radius. The Front-outside wheel has the largest turning radius (R_{Fo}) and should be used as reference.

A worksheet for Snake drive calculations was developed. For a 30” long x 20” drive-train, the results are:

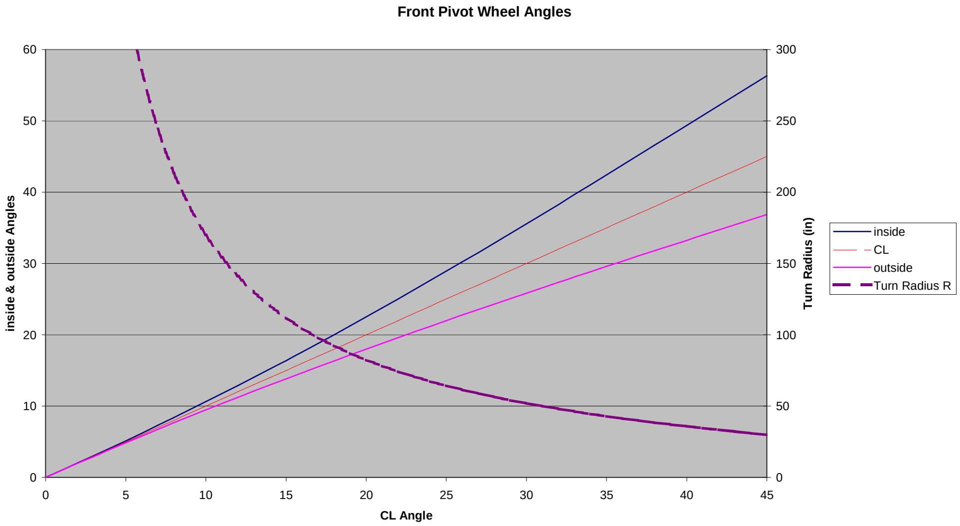
Pivot Wheel Robot - Automobile Drive Calculations

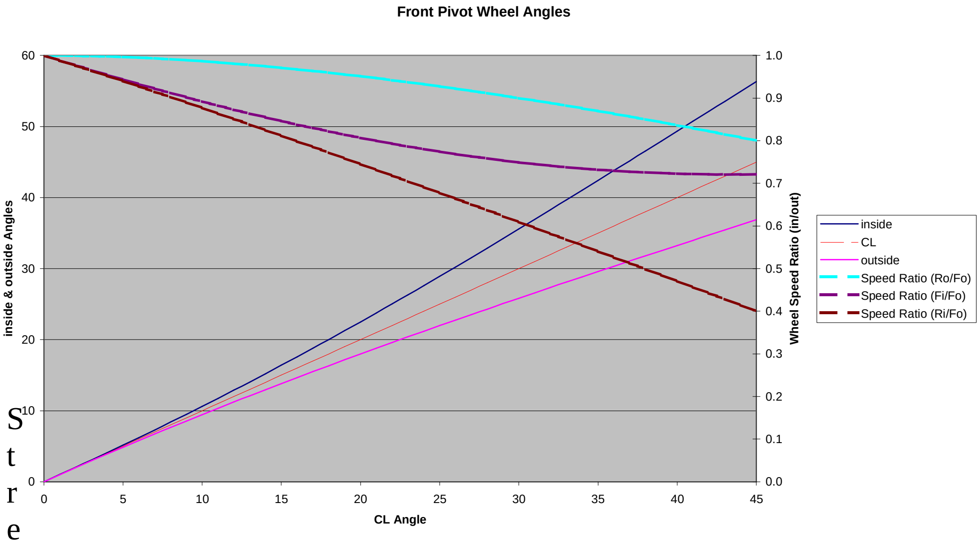
Robot Parameters

l	30	in
w	20	in

Turning angle (°)			Turn Radius (inches)					Speed Ratio, R_x/R_{Fo}		
inside	CL	outside	R_{Ri}	R_{Fi}	R_{RCL}	R_{Ro}	R_{Fo}	R_{Ro}/R_{Fo}	R_{Fi}/R_{Fo}	R_{Ri}/R_{Fo}
0.0	0.0	0.0	∞	∞	∞	∞	∞	1.000	1.000	1.000
5.1	5.0	4.9	332.9	334.3	342.9	352.9	354.2	0.996	0.944	0.940
10.6	10.0	9.5	160.1	162.9	170.1	180.1	182.6	0.986	0.892	0.877
16.4	15.0	13.8	102.0	106.3	112.0	122.0	125.6	0.971	0.846	0.812
22.5	20.0	18.0	72.4	78.4	82.4	92.4	97.2	0.951	0.807	0.745
28.9	25.0	22.0	54.3	62.1	64.3	74.3	80.2	0.927	0.774	0.678
35.6	30.0	25.8	42.0	51.6	52.0	62.0	68.8	0.900	0.749	0.610
42.4	35.0	29.6	32.8	44.5	42.8	52.8	60.8	0.870	0.732	0.541
49.4	40.0	33.3	25.8	39.5	35.8	45.8	54.7	0.836	0.723	0.471
56.3	45.0	36.9	20.0	36.1	30.0	40.0	50.0	0.800	0.721	0.400

Graphically:





Strengths:

- Efficient, hysteresis-free steering
- Responsive & intuitive
- Fine steering control – reduced small-angle steering sensitivity
- May be switched between x and y bias

Weaknesses:

- Not a 2d drive
- No turning in place possible

Tank *(Twitch)* Mode

Team 1640 built and tested a *Twitch* or Bi-axial Tank Drive (based on a 2008 robot built by Team 1565) prototype during the summer of 2008.

Tank Mode with a 4-wheel Pivot drive-train is functionally identical to *Twitch*.

From a control standpoint, Team 1640's *Twitch* prototype was set up so that any of the (4) robot sides could become the robot's front. Conceptually, a 1640 *Twitch* FRC robot could be built with different color bumpers on the (4) sides (or some other easily-recognized indicator). (4) buttons of the same colors (with lights or same colors, if possible) on the user interface would switch the front orientation (lights would indicate current front orientation).

Twitch is agile due to its ability to switch drive axis on the fly. Twitch steers well in y-bias, but cannot be steered in x-bias without using Omni-wheels.

The mathematics behind Tank Drive have already been established and are covered in Team 1640's Drive Lesson presentation.

Strengths versus the Pivot modes described above is not so clear. Strengths & Weaknesses of Tank and Pivot will be dealt with below.

Comparison – Tank Drive versus Pivot-Wheel Drive

Traditional Tank Drive is far and away the most commonly used drive approach used for FRC Robots. It is straightforward to design, build and maintain. It allows a common drive system for each side, providing available power to weighted wheels under uneven weight distributions, thereby optimizing traction.

Hitherto, all of Team 1640's robots have utilized tank drive:

1. Dewbot I (2005) 4wd x-bias with (4) 8" std wheels
2. Dewbot II (2006) rwd x-bias with (2) traction & (2) omni wheels
3. Dewbot III (2007) 4wd x-bias with (2) traction & (2) omni wheels
4. Dewbot IV (2008) rwd x-bias with (2) traction & (2) omni wheels
5. Dewbot V (2009) 6wd y-bias with (6) low-friction "Rover" wheels

Based on Team 1640's 2008 drive-train studies and the team's 2009 experience with Dewbot V, we understand that 6wd Tank Drive provides very adequate and reliable performance. It combines reasonable steering capabilities (excellent steering in y-bias) with good traction, mechanical simplicity, control simplicity and reliability.

So, why pursue Pivot-Wheel Drive?

Tank Drive's fundamental weakness is that in order to steer, you must slide wheels sideways (in the transverse direction) across a frictive surface. The reason 6wd Tank is superior to 4wd Tank is that it reduces this frictive turning resistance. The same is true for y-bias Tank versus x-bias Tank drive (therefore, the 6wd, y-bias Dewbot V is quite agile). Frictive resistance to turning creates steering hysteresis which leads to a tendency to over-steer and a loss of fine steering control. Frictive resistance also contributes to excessive tread wear.

Since Tank Drive robots must slide wheels to turn, it is imperative that enough torque be delivered to the wheels to overcome the loaded wheels' static frictive force ($F_f = \mu_s F_n$). This requirement doesn't exist with Pivot-Wheel Drive. Pivot Wheel Drive robots are therefore free to use lower gear reduction ratios and can potentially be faster than Tank Drive robots (with the same drive motors).

When maneuvering in a confined space, Tank Drive robots must take the time and space to rotate their chassis to change direction. This makes Tank Drive robots easier to block or pin. It also complicates and slows down alignment with ramps and other field and robot features. A robot having the capability of 2-d or bi-axial maneuverability possesses a competitive advantage under these situations.

A Pivot-Wheel Drive robot should provide hysteresis-free steering and therefore provide more accurate fine steering control.

Since there is no need to overcome frictive forces while turning, a Pivot-Wheel Drive robot (other factors being equal) should have greater turning torque available.

Pivot-Wheel Drive Snake, Automobile and Tank Modes all offer Bi-axial operation, while Crab Mode provides true 2-d maneuverability. This will make trapping or blocking a Pivot-Wheel robot more difficult (but not impossible, as our Chesapeake experience against 191 taught us). The ability to move sideways ought to simplify and speed up alignment with field and robot features (ramps, etc.).

6wd Tank Drive holds (or should hold) the advantage in:

- Simplicity of design and ease of execution
- Straightforward control
- Rugged
- Reliable
- Optimized traction under uneven wheel loading (including ramps)
- Eliminates the need for (4) steering motors/gearboxes/motor controllers/angle sensors/etc.
- Avoiding bottoming out at top of ramps
- Compatible with 2-speed gearbox
- Compatible with step climbing features

Pivot-Wheel Drive holds (or should hold) the advantage in:

- Improved fine steering control (no hysteresis)

- Should promote the practice of steering while moving (as opposed to move-stop-steer-move)
- Agility
- Speed (potentially) – you should be able to make a much faster Pivot-Wheel Drive robot using the same drive motors
- Avoiding entrapment & blockers
- Easier & faster alignment with field and robot features (such as ramps, outposts, airlocks, Trackballs, etc.)
- Improved ability to entrap & block opponents
- Unpredictability (to opponents)
- Reduced tread wear

In the final analysis, neither of these approaches is clearly superior to the other. Both possess strengths and weaknesses (or costs). A reasoned choice will need to be made after the competition objectives and constraints are known and a competition strategy developed. Having the knowledge and capability to build (and program) either at the start of build season improves a team's competitive position.

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[i] Vince Wilczynski and Stephanie Slezycki, FIRST Robots: Rack 'n' Roll: Behind the Design, Rockport (2008), pp 20-27